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ORIGINAL ARTICLE

Global optimal policy for vendor–buyer integrated inventory system within just in time environment

P. C. Yang · H. M. Wee · H. J. Yang

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Abstract Traditionally, inventory problems for the vendor and the buyer are treated separately. In modern enterprises, however, the integration of vendor–buyer inventory system is an important issue. This co-operative approach to inventory management contributes to the success of supply chain management by minimizing the joint inventory cost. The joint inventory cost and the response time can further be reduced when the buyer orders and the vendor replenishes the required items just in time (JIT) for their consumption. The inclusion of the JIT concept in this model contributes significantly to a joint inventory cost reduction. A numerical example and sensitivity analysis are carried out. The derived results show an impressive cost reduction when compared with Goyal's model.

Keywords Lot sizing · Just in time · Vendor–buyer integration

1 Introduction

In the past, economic order quantity (EOQ) and economic production quantity (EPQ) were treated independently from the viewpoints of the buyer or the vendor. In most cases, the optimal solution for one player was non-optimal to the other player. In today's competitive markets, close cooperation between the vendor and the buyer is

P. C. Yang (✉)
Industrial Engineering and Management Department, St. John's University, Tamsui, Taipei 25135, Taiwan, ROC

H. M. Wee
Industrial Engineering Department, Chung Yuan Christian University, Chungli 32023, Taiwan ROC

H. J. Yang
Aeronautical Engineering Department, Chung Cheng Institute of Technology, National Defense University, Taoyuan 335, Taiwan, ROC

necessary to reduce the joint inventory cost and the response time of the vendor–buyer system. The successful experiences of National Semiconductor, Wal-Mart, and Procter and Gamble have demonstrated that integrating the supply chain has significantly influenced the company's performance and market share (Simchi-Levi et al., 2000). Other studies (Weng 1995; Li et al. 1996; Yang and Wee 2000; Chen et al. 2001) show that an integrated approach results in improved performance and increased profitability to all players in the supply chain.

The idea of vendor–buyer integration has been studied in the sixties by Clark and Scarf (1960) who considered multi-echelon inventory-distribution systems. Banerjee (1986) derived a joint economic lot size model for a single vendor with finite production rate. Goyal (1988) extended Banerjee's model by relaxing the lot-for-lot production assumption. Ha and Kim (1997) derived the JIT system between the vendor and the buyer using geometric programming. Wee and Jong (1998) developed the JIT system between multiple parts and the finished product. A comprehensive review of vendor–buyer integration models was done by Goyal and Gupta (1989) and Thomas and Griffin (1996). Our study incorporated the JIT concept, and modified Goyal's model to develop a significant cost reduction approach to the problem.

2 Mathematical modeling and analysis

The mathematical model is developed on the basis of the following assumptions:

(a) Both the production and demand rates are constant.
(b) The integrated system of single-vendor and single-buyer is considered.
(c) The vendor and the buyer have complete knowledge of each other's information.
(d) Shortage is not allowed.

The decision variables are

- Q Buyer's lot size per delivery
- n Number of deliveries from the vendor to the buyer per vendor's replenishment interval.

The other related parameters are

- S Vendor's setup cost per setup
- A Buyer's ordering cost per order
- C_{v} Vendor's unit production cost
- C_{b} Unit purchase cost paid by the buyer
- r Annual inventory carrying cost per dollar invested in stocks
- P Annual production rate
- D Annual demand rate
- TC Integrated total cost of the vendor and the buyer when both the vendor and the buyer collaborate instead of being independent.

The vendor's time-weighted inventory, the product of inventory by time, is shown by gray color surrounded by the bold lines in Fig. 1. The buyer's replenishment interval is Q/D. The vendor's replenishment interval is nQ/D. The buyer's average inventory level is $Q/2$. The vendor's average inventory level I_{v} is derived as follows:

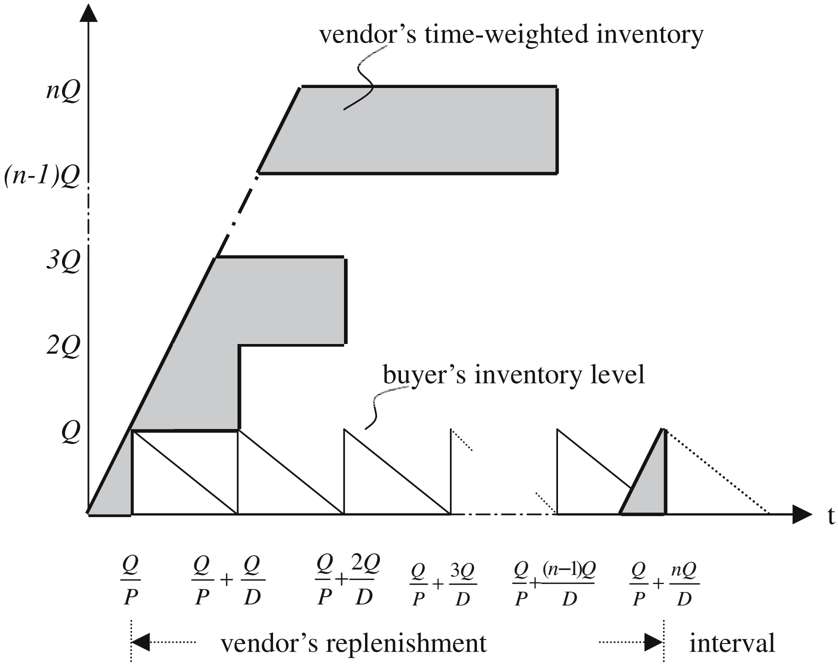

Fig. 1 Buyer's inventory level and vendor's time-weighted inventory

$$
\begin{aligned}
I_{\mathrm{v}} &= \frac{\text{vendor's time-weighted inventory}}{\text{vendor's replenishment interval}} \\
&= \frac{\dfrac{nQ^2}{2P} + Q^2\left(\dfrac{1}{D} - \dfrac{1}{P}\right) + 2Q^2\left(\dfrac{1}{D} - \dfrac{1}{P}\right) + \cdots + (n-1)Q^2\left(\dfrac{1}{D} - \dfrac{1}{P}\right)}{nQ!/!D}, \\
&= \frac{D}{nQ}\left[\frac{nQ^2}{2P} + \frac{n(n-1)Q^2}{2}\left(\frac{1}{D} - \frac{1}{P}\right)\right], \\
&= \frac{Q}{2}\left[(n-1)\left(1 - \frac{D}{P}\right) + \frac{D}{P}\right]. \qquad (1)
\end{aligned}
$$

The result in (1) is different from that of Goyal's model (1988) due to different modeling strategy. Goyal's model supplies goods to the buyer only after the termination of the production period, while our method supplies goods to the buyer as soon as there is enough to make up the batch-size, thus reducing the inventory cost during the production period (see Table A1, Appendix A).

Using (1), the integrated total cost of the vendor and the buyer per year is

$$
\mathrm{TC} = \frac{DA}{Q} + \frac{rQC_{\mathrm{b}}}{2} + \frac{DS}{nQ} + \frac{rQC_{\mathrm{v}}}{2}\left[(n-1)\left(1 - \frac{D}{P}\right) + \frac{D}{P}\right]. \qquad (2)
$$

In the right-side of (2), the first two terms are the buyer's annual ordering cost, and carrying cost, respectively, and the last two terms are the vendor's annual setup cost and carrying cost, respectively.

Equating the first derivative of (2) with regard to Q to zero and solving the equation, the buyer's economic lot size is

$$Q^* = \sqrt{\frac{2D\left(A + \frac{S}{n}\right)}{r\left\{C_\mathrm{b} + C_\mathrm{v}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]\right\}}}. \tag{3}$$

After substituting (3) into (2), the optimal integrated total cost is

$$\mathrm{TC}^* = \sqrt{2Dr\left(A+\frac{S}{n}\right)\left[C_\mathrm{b} + C_\mathrm{v}\left((n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right)\right]}. \tag{4}$$

Substituting (3) into (2), equating the first derivatives with respect to n to zero and solving the equation, the value of n that minimizes the integrated total cost is

$$n = \sqrt{\frac{S\left[C_\mathrm{b} - C_\mathrm{v}\left(1-\frac{2D}{P}\right)\right]}{AC_\mathrm{v}\left(1-\frac{D}{P}\right)}}. \tag{5}$$

Since the value of n is positive integer, the optimal value of n, denoted by n^*, is

$$n_1^* = \left\lfloor \sqrt{\frac{S\left[C_\mathrm{b} - C_\mathrm{v}\left(1-\frac{2D}{P}\right)\right]}{AC_\mathrm{v}\left(1-\frac{D}{P}\right)}} \right\rfloor, \quad \text{when } \mathrm{TC}(n_1^*) \le \mathrm{TC}(n_1^*+1) \tag{6}$$

or

$$n_2^* = \left\lfloor \sqrt{\frac{S\left[C_\mathrm{b} - C_\mathrm{v}\left(1-\frac{2D}{P}\right)\right]}{AC_\mathrm{v}\left(1-\frac{D}{P}\right)}} \right\rfloor + 1, \quad \text{when } \mathrm{TC}(n_2^*) \le \mathrm{TC}(n_2^*-1), \tag{7}$$

where $\lfloor x \rfloor$ is the greatest integer less than or equal to x, and $TC(n)$ represents that the TC is a function of n.

From (3), the economic lot size of the lot-for-lot condition (i.e., when $n = 1$) is

$$Q^* = \sqrt{\frac{2D\,(A+S)}{r\left[C_\mathrm{b} + C_\mathrm{v}\left(\frac{D}{P}\right)\right]}}. \tag{8}$$

The result of (8) is the same as Banerjee's model (1986).

From (3), when the production rate is very large, the economic lot size becomes

$$Q^* = \sqrt{\frac{2D\left(A+\frac{S}{n}\right)}{r\left[C_\mathrm{b} + (n-1)C_\mathrm{v}\right]}}. \tag{9}$$

The result of (9) is the same as (12.7) of Silver et al. (1998).

3 Numerical example

The model developed in the preceding section can be illustrated by the numerical example of Goyal (1988) as follows:

Table 1 Summary of computational results

Models	Goyal's model	Our model
n	2	5
Buyer's order size	198	110
Vendor's lot size	396	550
Buyer's annual cost	\$621.3	\$502.4
Vendor's annual cost	\$1,653.6	\$1,400.9
Integrated total cost	\$2,274.9	\$1,903.3

Table 2 Sensitivity analysis

Parameter	Change	Integrated total cost by Goyal (A)	Integrated total cost of this study (B)	PTCR $= \left(\frac{A-B}{A}\right)$
D	+30%	\$2,677.0	\$2,098.1	21.6%
D	−30%	\$1,840.2	\$1,633.6	11.2%
P	+30%	\$2,217.1	\$1,941.5	12.4%
P	−30%	\$2,378.5	\$1,809.4	23.9%
A	+30%	\$2,312.5	\$1,960.2	15.2%
A	−30%	\$2,236.6	\$1,834.1	18.0%
S	+30%	\$2,560.3	\$2,109.6	17.6%
S	−30%	\$1,948.1	\$1,659.8	14.8%
r	+30%	\$2,593.7	\$2,170.1	16.3%
r	−30%	\$1,903.3	\$1,592.4	16.3%

PTCR Percentage total cost reduction of our model as compared with Goyal's model

Annual demand rate, $D = 1000$.
Annual production rate, $P = 3200$.
Buyer's ordering cost, $A = \$25$ per order.
Vendor's setup cost, $S = \$400$ per setup.
Buyer's unit purchase cost, $C_{\mathrm{b}} = \$25$.
Vendor's unit production cost, $C_{\mathrm{v}} = \$20$.
Annual inventory carrying cost per dollar, $r = 0.2$.

From (5), the value of n is equal to 4.5126.

Since $\mathrm{TC}(\text{if } n = 5) - \mathrm{TC}(\text{if } n = 4) = 1903.3 - 1903.9 < 0$, the optimal number of deliveries is 5. By applying the above solution procedure, the computational results of the two models (Goyal's and our model) are given in Table 1. Sensitivity analysis is carried out by increasing or decreasing the individual parameter by 30% with all other parameters remaining unchanged. By comparing our model with Goyal's, the percentage total cost reduction (PTCR) is derived in Table 2. The range of PTCR is found to be from 11.2 to 23.9%, with an average value of about 16.8%.

4 Concluding remark

This study develops an economic lot size policy for an integrated vendor–buyer system. Both the joint inventory cost and the response time are improved by implementing JIT, which shortens delivery time cycle and reduces the inventory cost. The integrated total cost of our model is shown to be lower than that of Goyal's model. This is because the vendor in our model supplies goods to the buyer as soon as there is enough to make-up the batch-size. When we compare our result with Goyal's model, a significant percentage total cost reduction of about 11.2–23.9% is achieved.

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Appendix A: Comparison of models

The two models are compared using the example from Goyal (1988).

Annual demand rate, $D = 1000$.
Annual production rate, $P = 3200$.
Buyer's ordering cost, $A = \$25$ per order.
Vendor's setup cost, $S = \$400$ per setup.
Buyer's unit purchase cost, $C_{\rm b} = \$25$.
Vendor's unit production cost, $C_{\rm v} = \$20$.
Annual inventory carrying cost per dollar, $r = 0.2$.

The computational results of the two models are given in Table A1.
The vendor's average inventory from Goyal's model is

$$I_{\rm v} = \frac{\frac{n^2Q^2}{2P} + Q^2\left(\frac{n-1}{D} + \frac{n-2}{D} + \cdots + \frac{1}{D}\right)}{nQ/D}, \\ = \frac{Q}{2}\left(n\left(1+\frac{D}{P}\right)-1\right). \tag{10}$$

From (1), the vendor's average inventory from our model is

$$I_{\rm v} = \frac{Q}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]. \tag{11}$$

Table A1 Comparison of the two models

Models	Goyal's model (1988)	Our model
$I_{\rm v}$	$\frac{Q}{2}\left(n\left(1+\frac{D}{P}\right)-1\right)$	$\frac{Q}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]$
TC	$\frac{DA}{Q}+\frac{QrC_{\rm b}}{2}+\frac{DS}{Q}+rC_{\rm v}I_{\rm v}$	$\frac{DA}{Q}+\frac{QrC_{\rm b}}{2}+\frac{DS}{Q}+rC_{\rm v}I_{\rm v}$
Q^*	$\sqrt{\frac{2D\left(A+\frac{S}{n}\right)}{r\left[C_{\rm b}-C_{\rm v}+nC_{\rm v}\left(1+\frac{D}{P}\right)\right]}}$	$\sqrt{\frac{2D\left(A+\frac{S}{n}\right)}{r\left\{C_{\rm b}+C_{\rm v}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]\right\}}}$
TC(if $n = 1$)	\$2,304.9	\$2,304.9
TC(if $n = 2$)	\$2,274.9*	\$2,012.5
TC(if $n = 3$)	\$2,303.1	\$1,928.9
TC(if $n = 4$)	\$2,345.2	\$1,903.9
TC(if $n = 5$)	\$2,392.2	\$1,903.3*
TC(if $n = 6$)	\$2,441.0	\$1,914.9

Q^* Buyer's economic lot size

The decrease in the vendor's average inventory using our model is $\frac{QD}{P}(n-1)$ that is positive for integer greater than zero. Thus, we can see that our model is better than Goyal's model.

References

Banerjee, A.: A joint economic lot size model for purchaser and vendor. Decis. Sci. **17**, 292–311 (1986)

Chen, F., Federgruen, A., Zheng, Y.S.: Coordination mechanisms for a distribution system with one supplier and multiple retailers. Manage. Sci. **47**(5), 693–708 (2001)

Clark, A.J., Scarf, H.: Optimal policies for a multi-echelon inventory problem. Manage. Sci. **6**, 475–490 (1960)

Goyal, S.K.: A joint economic lot size model for purchaser and vendor: A comment. Decis. Sci. **19**, 236–241 (1988)

Goyal, S.K., Gupta, Y.P.: Integrated inventory models: the buyer-vendor coordination. Euro. J. Oper. Res. **41**, 261–269 (1989)

Ha, D., Kim, S.L.: Implementation of JIT purchasing: an integrated approach. Prod. Plan. Control **8**, 152–157 (1997)

Li, S.X., Huang, Z., Ashley, A.: Inventory, channel coordination and bargaining in a manufacturer-retailer system. Ann. Oper. Res. **68**, 47–60 (1996)

Silver, E.A., Pyke, D.F., Peterson, R.: Inventory Management and Production Planning and Scheduling. Wiley Inc, New York (1998)

Simchi-Levi, D., Kaminsky, P., Simchi-Levi, E.: Designing and Managing the Supply Chain. McGraw-Hill Companies, Singapore (2000)

Thomas, D.J., Griffin, P.M.: Coordinated supply chain management. European J. Oper. Res. **94**, 1–15 (1996)

Wee, H.M., Jong, J.F.: An integrated multi-lot-size production inventory model for deteriorating items. Manage. Syst. **5**, 97–114 (1998)

Weng, Z.K.: Channel coordination and quantity discounts. Manage. Sci. **41**(9), 1509–1522 (1995)

Yang, P.C., Wee, H.M.: Economic ordering policy of deteriorated item for vendor and buyer: an integrated approach. Prod. Plan. Control 11(5), 474–480 (2000)